

NAVAL POSTGRADUATE SCHOOL

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THESIS

**AN OPTIMIZATION TECHNIQUE USING THE
FINITE ELEMENT METHOD AND ORTHOGONAL
ARRAYS**

by

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In order to meet the objective, an optimization procedure was developed by coupling the finite element analysis (FEA) to the orthogonal array experimentation technique, because FEA is a common analysis tool for design engineers. From the results of the FEA and an orthogonal array, an average Jacobian matrix was constructed that showed the average overall sensitivity of the design variables. These sensitivities were then used to optimize the design parameters. The process could then be repeated at the discretion of the engineer until a satisfactory design is obtained. In general, the designer can predict and control the number of FEA calculations before an optimization process so that one can plan a budget and time for an optimal design.

Some examples of structural optimization with truss and frame structures with continuous and discrete values of design variables were studied using the technique developed in this paper. Their optimal solutions were found with a small number of iterations.

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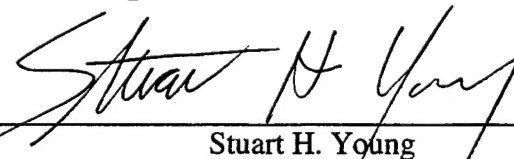
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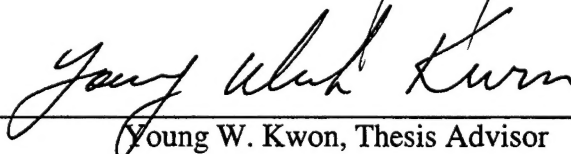
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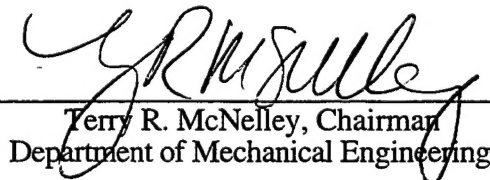
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I. INTRODUCTION

Optimization is a process that can be seen in almost every aspect of life. Humans are always striving to go faster, climb higher, run farther, and engineering is no different. Engineering requires optimization so that we can design faster planes and cars, that use less fuel, are lighter, stronger, and more comfortable. Engineers have been optimizing designs since the beginning, but recent advances in computing have made numerical optimization techniques a more effective way than the original trial-and-error and experience based optimization. The present methods based on numerical search techniques are effective and easy to understand and implement on design problems that don't have many design variables (Vanderplaats, 1984). However, it is difficult to estimate the computational time and budget to converge to an optimal design using most of the numerical optimization techniques available today unless the design engineer has experience with the same kind of design problem in advance. The computational costs increase highly nonlinearly as the number of design variables increases. Therefore, application of a numerical optimization technique to a practical engineering design has been hampered.

The objective of this research was to develop an efficient method for achieving the optimal design with minimal computational effort. Also, the developed technique allows the engineer to interact with the procedure and to estimate how much time is to be spent on the optimization depending on the number of design variables (it does not matter how many design variables there are), and how close to the actual optimum the design must be, balancing schedule and cost. The approach was to develop an optimization procedure by combining the finite element method and orthogonal arrays. The emphasis was in structural mechanics, but could be applied to other disciplines. Because finite element analyses are time consuming in general, the approach was to minimize the number of iterations

associated with the FEA. This led to the incorporation of orthogonal arrays in the methodology in order to reduce the number of FEA computations.

Orthogonal arrays were developed initially for discrete values of design variables. Therefore, they work well for the discrete case. Further, the optimization technique was also applied to continuous design variables with design constraints.

The goal is not necessarily to get the actual optimum value, assuming that one exists, but to develop a more practical approach that will achieve a result close to the optimum with reasonable effort. In addition to developing a methodology that would lead to an optimal design, it was necessary to identify the overall sensitivity of the various parameters. This information provides an engineer with further physical understanding of the design problem.

Although the technique may not achieve the most optimal value, it will give the designer the value of parameters close to the optimum with reasonable effort and drastically reduced computational time. Just like if you want a higher quality car you might buy a luxury car, or a faster car you might buy a sports car, you must pay the price. Similarly, if you want the most optimum design you must pay for it with either higher design and manufacturing costs or more computer time. On the other hand, if you are willing to accept a good solid car you might buy a basic family car. This is similar to spending a reasonable amount of time optimizing and getting a close to optimal design without the high computer computational times. The latter approach is therefore the goal which should lead to a more practical optimization approach.

This paper will first discuss the background of the Taguchi Method and the orthogonal arrays that he has incorporated into his design of experiments methodology. This will be followed by a brief technical description of the orthogonal arrays as described by Peace (1993). Next will be the method and procedure developed in this research. The final section will include some examples of the technique.

II. TAGUCHI METHOD AND ORTHOGONAL ARRAYS

The optimization approach developed in this research centered around the concepts developed by Taguchi for quality engineering and then coupling his concepts with the finite element method to achieve an optimal design methodology. The following brief description of orthogonal arrays can be found in Peace (Peace, 1993).

The orthogonal array was the foundation of the methodology that Taguchi developed for designing an experiment. Although other methods of conducting an experiment are possible, like a full factorial approach, the orthogonal array has traditionally been associated with the Taguchi experimentation technique. The orthogonal array is repeatable, cost effective, and efficient in that only a small amount of data must be collected and then that data can easily be translated into meaningful and verifiable conclusions (Peace, 1993).

The orthogonal array was originally developed by Fisher of England. It was developed during his efforts to control error in an experiment (Ealey, 1988). Later, Taguchi adapted the orthogonal array in order to measure the effect of a factor being studied on the average result (Peace, 1993).

The details of the orthogonal array will follow, but the concept of orthogonality refers to the statistically independent or balanced parameters that make up the columns of the orthogonal array. One major objective of this research was the application of the technique in both discrete and continuous parameter problems. However, the orthogonal array approach lends itself directly only to problems with discrete parameters. For problems associated with discrete parameters, the orthogonal array is used directly. But, for problems with continuous variables, this research will show that taking the minimum and maximum values in the range of each of the parameters, and then putting those values into the orthogonal array, is an effective way of dealing with continuous variables. A discussion of how this was dealt with will follow this section.

In a typical orthogonal array as shown in Table 1 (Taguchi, 1987), each level (here, 1 or 2) has an equal number of occurrences in each column. Also notice the relationship between one column and another. For each level within one column, each level within any other column will occur an equal number of times as well. This introduces the statistical independence or balance into the orthogonal array. Since each column is orthogonal to the others, if the results associated with one level of a specific factor are much different at another level, it is because changing that factor from one level to another has a strong impact on the design parameter being measured. Because the levels of the other factors are occurring an equal number of times, any effect by these other factors should be canceled out (Peace, 1993).

Table 1. $L_8(2^7)$ Orthogonal Array							
Parameter No.	$L_8(2^7)$						
	1	2	3	4	5	6	7
No. of Runs							
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

The design of an orthogonal array experiment, although balanced, does not require that all combinations of all factors be tested. The full factorial experiment studying 7 factors would require 128 ($= 2^7$) experimental runs, compared with the 8 runs in the orthogonal array. The advantage of the orthogonal array can be seen in its simplicity, time savings, and therefore cost savings.

It should be noted, that the columns in the orthogonal array can represent not only design factors, but also interactions. Therefore, by using sound engineering judgment, the orthogonal array can include critical interactions. The orthogonal array, does not study all interaction combinations like a full factorial experiment, but the balanced nature of the orthogonal array can compensate for this.

The terminology of the orthogonal array $L_A(B^C)$ is as follows. The subscript of L, which here is A, represents the number of experimental runs which can be conducted in the experiment. B denotes the number of levels within each column, and C denotes the number of columns (factors and/or interactions that can be studied) in the orthogonal array. Therefore, the above orthogonal array $L_8(2^7)$ contains eight experimental runs. Each column contains '2' levels, meaning each factor will possess two levels. The '7' means that up to seven factors and interactions can be incorporated into the experiment.

III. METHOD & PROCEDURE

As discussed earlier, the procedure that has been developed focuses on the orthogonal array. The following example is written for a structural optimization problem with stress, displacement, natural frequencies, etc. as the design constraints, but in general could be applied to other disciplines. The design variables varied from a minimum value to a maximum value continuously or discretely. The objective was to reduce the weight of the structure supporting a load while not exceeding the design constraints in any of the members of the structure. In this example, the cross-sectional areas are considered as the design variables, the weight of the structure was the objective function, and the stresses, displacements, and natural frequencies were the constraints.

The general procedure of the optimization technique that was developed begins by constructing the proper orthogonal array for the given design variables using the entire range of values for each variable. If the design variables are continuous, then the values of 1 and 2 are used in the orthogonal array corresponding to the minimum and maximum values respectively. If the design variables are discrete, then the lowest discrete value is used for level 1 in the orthogonal array, the next lowest value is used for level 2 in the orthogonal array, etc.

The next step is to use the design values in each row of the orthogonal array and run a FEM program (Kwon, 1997) for each row using the corresponding design values in the columns of the orthogonal array. The inputs to the FEM code are the cross-sectional areas, and the output from the FEM code are the values of normalized stress, displacements, natural frequencies, etc., P_i . Here P_i is the constraint function and is defined as

$$P_i = \frac{k_i f_i}{S_i} \leq 1 \quad (1)$$

where f_i is a computed value from the finite element analysis such as stress, displacement, or natural frequency. And, S_i is the allowable value of f_i such as failure strength, displacement, or natural frequency, and k_i is a factor of safety ($k_i \geq 1$).

After all cases have been run in the orthogonal array, the results are plotted P_i vs. x_j which is the sensitivity of a normalized constraint value with respect to a design variable, namely, the average overall sensitivities (slopes) $\frac{\Delta P_i}{\Delta x_j}$ are calculated. With the desired value for all $P_i = 1$, a target value of \bar{P}_i was selected using a one-dimensional (1-D) search technique in the direction determined by the average overall sensitivities until a constraint boundary was encountered.

The next step is to take the best value from all of the orthogonal array runs that meet the design criteria and constraints, although it will be shown that the final solution is relatively independent of the starting point. It is recommended that the starting point be in the feasible domain for all parameters, because the technique is attempting to approach the optimum design from within the feasible domain. If no cases in the orthogonal array results are feasible, the most conservative design may be used as the initial design. These cross-sectional areas will then form the vector, $\{x^j\}$. These values are then used to run the FEA code, which produces a vector of P-values called $\{P^j\}$. The superscript denotes the iteration number.

From the orthogonal array, the average overall sensitivity of each parameter over its entire domain is calculated. This is slightly different from the common optimization approach of calculating the Jacobian (which contains the local sensitivities calculated at one point). In the traditional approach, the Jacobian matrix is determined with a fixed value of each design variable. The Jacobian matrix (Lang, 1968) is

$$[J] = \begin{bmatrix} \frac{dP_1}{dx_1} & \frac{dP_1}{dx_2} & \dots & \frac{dP_1}{dx_n} \\ \frac{dP_2}{dx_1} & \frac{dP_2}{dx_2} & \dots & \frac{dP_2}{dx_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{dP_m}{dx_1} & \frac{dP_m}{dx_2} & \dots & \frac{dP_m}{dx_n} \end{bmatrix} \quad (2)$$

where n is the number of design variables and m is the number of constraints. In the present study, the averaged Jacobian matrix over the domain of each variable can be assembled from the average overall sensitivities instead of being computed at a specific set of design values. Thus the averaged Jacobian matrix is

$$[\bar{J}] = \begin{bmatrix} \frac{\Delta P_1}{\Delta x_1} & \frac{\Delta P_1}{\Delta x_2} & \dots & \frac{\Delta P_1}{\Delta x_n} \\ \frac{\Delta P_2}{\Delta x_1} & \frac{\Delta P_2}{\Delta x_2} & \dots & \frac{\Delta P_2}{\Delta x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\Delta P_m}{\Delta x_1} & \frac{\Delta P_m}{\Delta x_2} & \dots & \frac{\Delta P_m}{\Delta x_n} \end{bmatrix} \quad (3)$$

In general, the off-diagonal terms in the first n rows are small compared with the diagonal terms and are therefore neglected. Thus $[\bar{J}]$ can be rewritten as

$$[\bar{J}] = \begin{bmatrix} \frac{\Delta P_1}{\Delta x_1} & 0 & \dots & 0 \\ 0 & \frac{\Delta P_2}{\Delta x_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\Delta P_n}{\Delta x_n} \\ \frac{\Delta P_{n+1}}{\Delta x_1} & \frac{\Delta P_{n+1}}{\Delta x_2} & \dots & \frac{\Delta P_{n+1}}{\Delta x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\Delta P_m}{\Delta x_1} & \frac{\Delta P_m}{\Delta x_2} & \dots & \frac{\Delta P_m}{\Delta x_n} \end{bmatrix} \quad (4)$$

Out of the terms in the averaged Jacobian matrix, the absolute maximum in each column is used because that value corresponds to the most constraining value when the design variables are updated. Using these sensitivities, the following equation is solved to compute the change of the design variables

$$\Delta P_i = \left(\frac{\Delta P_i}{\Delta x_j} \right)_{\max} \Delta x_j \quad (5)$$

for $i=1\dots n$ and $j=1\dots m$. Here Δx_i is simply the needed change in each design variable to achieve an optimal value which is to be solved for using

$$\Delta x_j = \Delta P_i \left(\frac{\Delta P_i}{\Delta x_j} \right)_{\max}^{-1} \quad (6)$$

where

$$\Delta P_i = \{\bar{P}_i - P_i\} \quad (7)$$

Here, ΔP_i is the distance to be traveled in the direction determined by the orthogonal array from the current value of P_i to the target value \bar{P}_i . The target value \bar{P}_i is the value to be determined from the 1-D search. This value can be found by extrapolation until a constraint is violated and then interpolated until the constraint boundary is found. Many 1-D search techniques exist that can efficiently accomplish this task such as the bisection method or the golden section method (Vanderplaats, 1984).

Δx_j is then added to the previous design value x_j^1 to yield the new design values x_j^2 . The new design values are then used in running the FEA program again to verify the result.

At this point, if any or all of the values in P_j^2 are close to 1, then this is assumed to be very close to the optimum design, and the analysis can be stopped, with the design values becoming x_j^2 . If further reduction is required, then the range of the parameter values is refined by using $\pm 8\%$ of the values in x_j^2 . The procedure is then repeated using the new refined maximum and minimum values as the limits in the orthogonal array until a satisfactory design is obtained. Usually, more than one set of orthogonal arrays is not necessary unless a very refined optimal design is required. This further refinement is the topic of follow-on studies.

For a discrete value of a design variable, the change of design variable Δx_j is compared to the allowable change of the design variable. If the magnitude of the calculated Δx_j is greater than the magnitude of the allowable change, then Δx_j is set equal to the allowable value. Otherwise, the design variable does not change. This process continues until there is no longer any change in the design.

Figure 1 outlines the procedure described above.

Optimization Procedure using Orthogonal Arrays

- | |
|---|
| <ol style="list-style-type: none">1. Construct the Proper Orthogonal Array.2. Select an Initial (Feasible) Design.3. Calculate the Average Overall Sensitivities.4. Update the Design Variables using a 1-D Search Technique.5. If the design is satisfactory, then stop the procedure. Otherwise, repeat the procedure with a refined domain for further optimization. |
|---|

Figure 1. Procedure Summary

IV. RESULTS & DISCUSSION

In this section, three examples will be shown to illustrate the technique described in the previous section. The first example is a statically independent 3-bar truss with continuous design variables and stress/displacement constraints. The second example is a statically independent 6-bar truss in a star shape with discrete design variables and only stress constraints. The third example is a 12-bar frame structure with continuous design variables. It has stress, maximum displacement, and minimum natural frequency constraints. The frame problem in the third example is first solved using a conventional numerical optimization technique, Sequential Quadratic Programming (SQP). Next, the frame problem is solved with orthogonal arrays using the techniques developed in this research. Finally, the results of the two optimization techniques are compared.

A. STATICALLY INDETERMINATE THREE-BAR TRUSS EXAMPLE

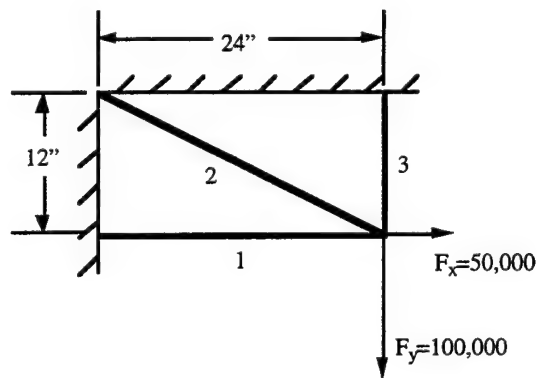


Figure 2. 3-Bar Truss

This first example is a statically independent three-bar truss structure subjected to a load of 50,000 lbs to the right and 100,000 lbs down. The objective is to minimize the weight of the structure such that each member does not exceed the failure strength of 118 ksi

and the displacement at the node where the load is applied should not exceed 0.12 inches horizontally and -0.06 inches vertically. A factor of safety of 1.5 was used for the stress and displacement values. Initially, the minimum cross-sectional area is 0.25 in², and the maximum cross-sectional area is 2.00 in² for all members. The design variables (cross-sectional area) in this problem were continuous. These areas are then put into the following $L_4(2^3)$ orthogonal array (Phadke, 1989) (See Table 2).

Table 2. $L_4(2^3)$ Orthogonal Array

Run #	Area 1	Area 2	Area 3
1	0.25	0.25	0.25
2	0.25	2.00	2.00
3	2.00	0.25	2.00
4	2.00	2.00	0.25

Table 3 shows the results from running the FEM code for each of the 4 rows in the orthogonal array, Table 2.

Table 3. FEA Results

Run #	P1	P2	P3	P4	P5	WT
1	1.03	1.69	4.33	1.53	6.43	15.71
2	0.27	0.32	0.49	0.41	0.73	83.67
3	0.28	0.35	0.62	0.41	0.91	78.71
4	-0.15	0.52	3.22	-0.22	4.77	104.67

The first three columns (P1 - P3) are the normalized stresses and the next two columns (P4 - P5) are the normalized displacements of the node where the load is applied.

Figure 3 is a sensitivity plot for the normalized stress in element 1 with respect to the three different cross-sectional areas (A1, A2, A3). Similarly, Figure 4 is a sensitivity plot for the normalized stress in element 2 with respect to the three different cross-sectional

areas. And finally, Figure 5 is a sensitivity plot for the normalized stress in element 3 with respect to the three different cross-sectional areas. In all these three plots, the dotted lines represent the stress relative to element 1, the dashed lines represent the stress relative to element 2, and the dash-dot lines represent the stress relative to element 3. From these plots, one can see that the slope of the normalized stress in each member vs. the area of that same member is the steepest (or close to the steepest) and is therefore has the highest average overall sensitivity.

We then take only the sensitivity plot for element 1 with respect to the cross-sectional area of element 1 and plot this sensitivity in Figure 6. Similarly, Figure 7 is the sensitivity of element 2 with respect to the cross-sectional area in element 2. Finally, Figure 8 is for element 3. Notice that the slope in Figure 8 is the steeper than Figures 6 and 7, indicating that the stress level in element 3 is much more sensitive to changes in cross-sectional area 3 than in elements 1 and 2.

Figures 9 - 11 show the sensitivities of the normalized horizontal displacement, P_4 , to the three cross-sectional areas. Here we see that all of the slopes are about the same indicating that the horizontal displacement is affected equally by changes in all three elements.

Figures 12 - 14 show the sensitivities of the normalized horizontal displacement, P_4 , to the three cross-sectional areas. Figure 14 shows the steepest slope, indicating that changes in the cross-sectional area of element 3 will have the greatest impact on the vertical displacement.

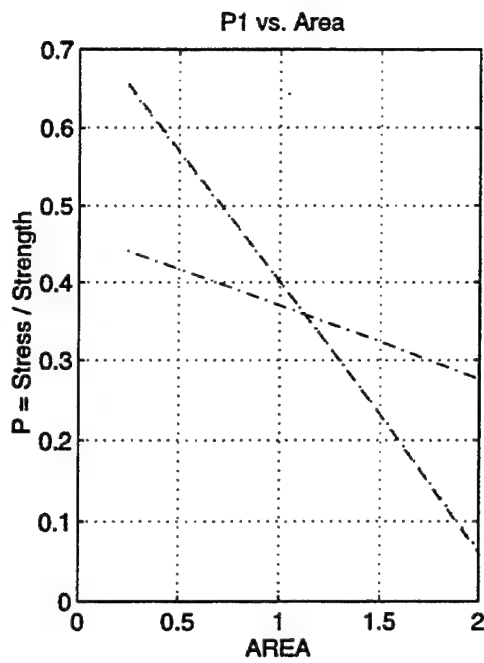


Figure 3. Sensitivity Plot, P1 vs. Area

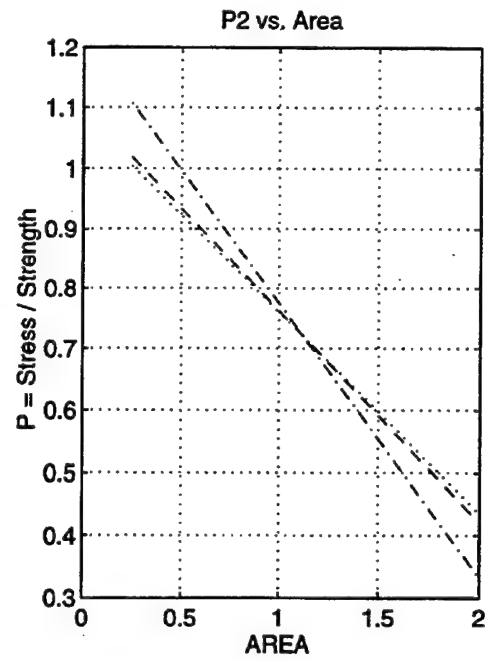


Figure 4. Sensitivity Plot, P2 vs. Area

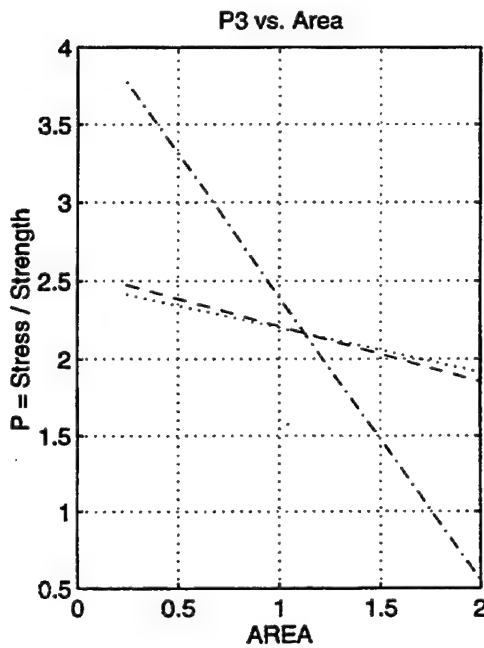


Figure 5. Sensitivity Plot, P3 vs. Area

LEGEND

Element 1 = (dotted line)

Element 2 = - - - - - (dashed line)

Element 3 = - . - . - . (dash-dot line)

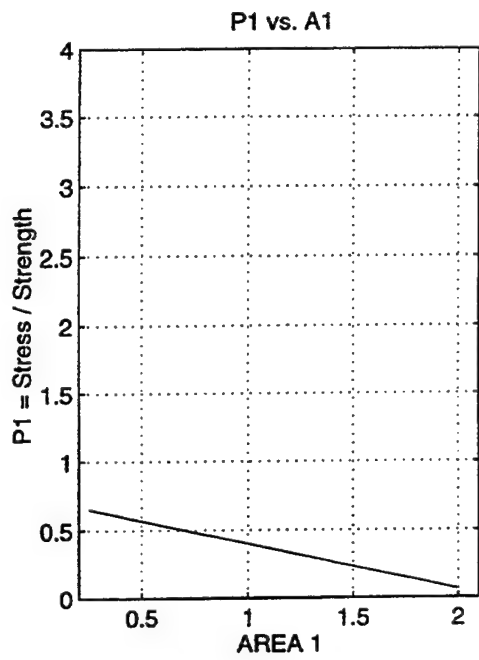


Figure 6. Sensitivity Plot, P1 vs. A1

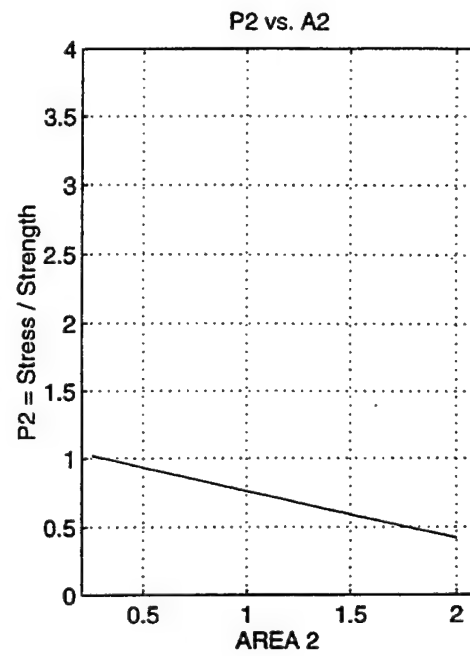


Figure 7. Sensitivity Plot, P2 vs. A2

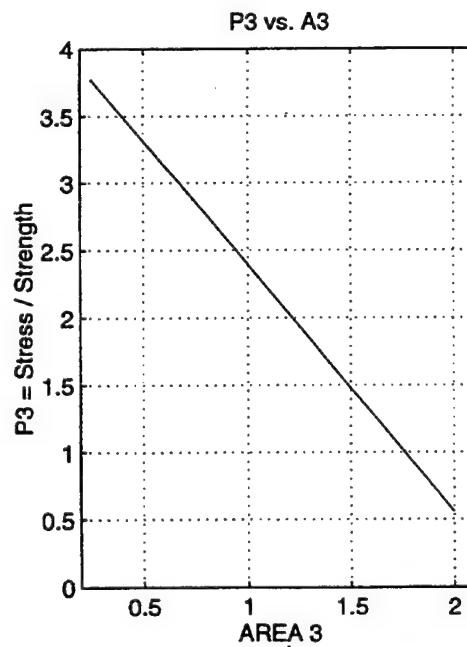


Figure 8. Sensitivity Plot, P3 vs. A3

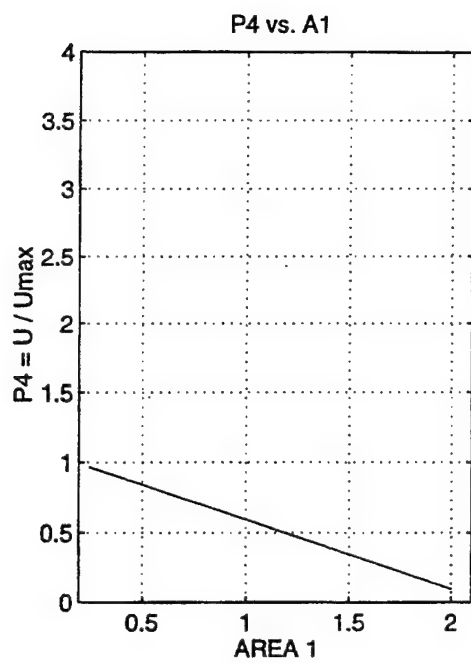


Figure 9. Sensitivity Plot, P4 vs. A1

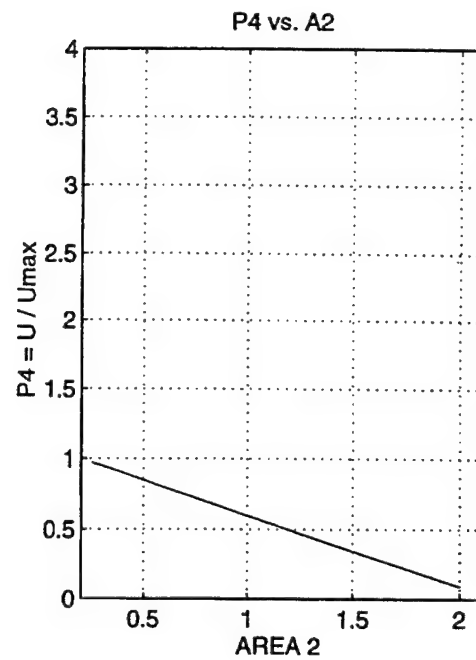


Figure 10. Sensitivity Plot, P4 vs. A2

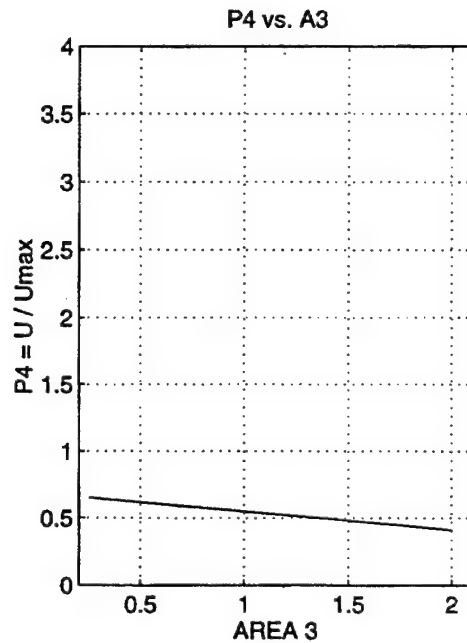


Figure 11. Sensitivity Plot, P4 vs. A3

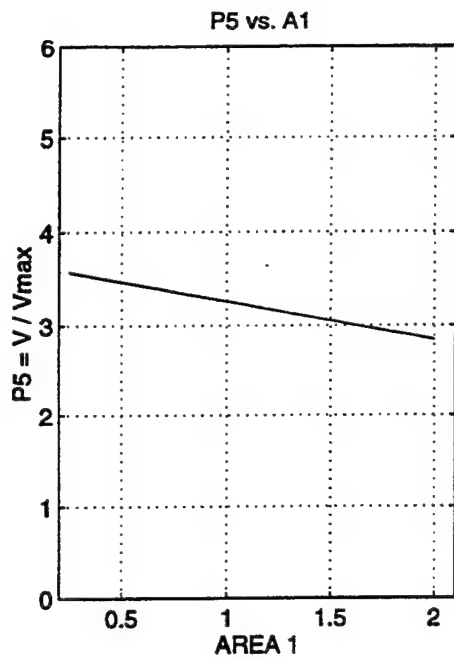


Figure 12. Sensitivity Plot, P5 vs. A1

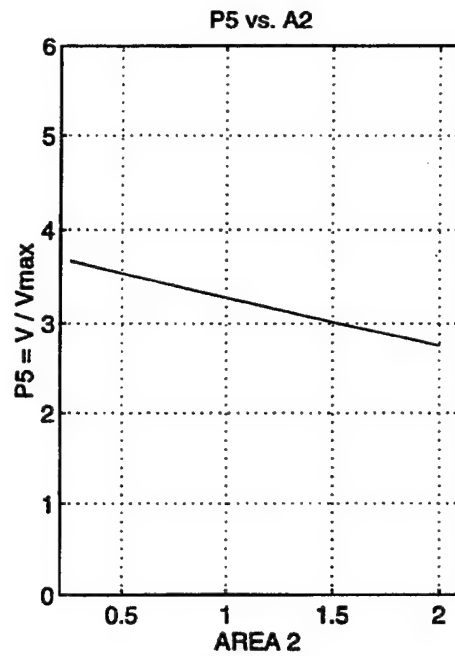


Figure 13. Sensitivity Plot, P5 vs. A2

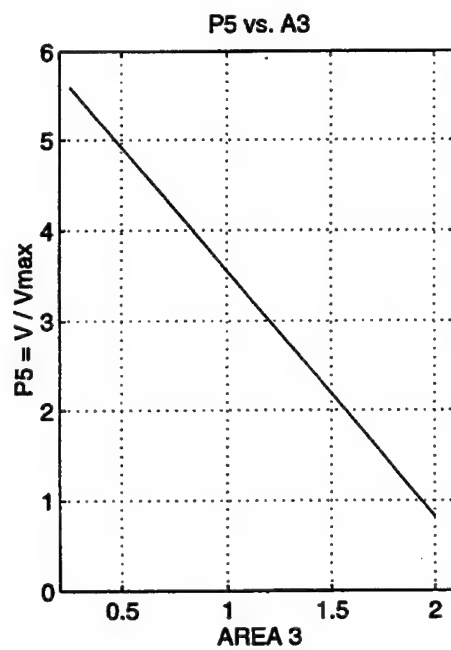


Figure 14. Sensitivity Plot, P5 vs. A3

In this analysis we will use run 3 as the starting point because it has the lowest weight of the runs where no constraints are violated. The starting point $\{A^1\}$ is

$$\{A^1\}^T = \{2.00 \quad 0.25 \quad 2.00\} \quad (8)$$

with the resulting

$$\{P^1\}^T = \{0.28 \quad 0.35 \quad 0.62 \quad 0.41 \quad 0.91\} \quad (9)$$

It meets the criteria that none of the constraints are violated, therefore the design variables are all in the feasible domain. This run results in a weight of 78.71 lbs. This becomes the $\{A^1\}$ vector with the corresponding result, $\{P^1\}$.

In order to optimize the design variables, the following calculations were performed for each member. For example, for member 1 equation (5) is applied for both stress and displacement constraints with a target value of 0.9. From this, three values (1 from stress constraint and 2 from the displacement constraints) of ΔA were calculated, then the largest absolute value was chosen to update the design variables. The resulting design variables were

$$\{A^2\}^T = \{0.75 \quad 0.25 \quad 1.90\} \quad (10)$$

In order to verify that all of the design constraints were met, the FEM code was run using these design variables, with the following results

$$\{P^2\}^T = \{0.65 \quad 0.65 \quad 0.63 \quad 0.97 \quad 0.94\} \quad (11)$$

The corresponding weight for this design is 47.55 lbs, a considerable weight reduction after only five FEA computations. The results show that the new design variables satisfy the constraints and are therefore an acceptable solution. However, if the engineer wants to further optimize the design variables, then the procedure can be repeated with a refined domain.

The refining of the domain is done individually for each cross-sectional area. $\{A_{MIN}\}$ is calculated from the equation

$$\{A_{MIN}^j\} = \{A^{j-1}\} - \beta \left\{ (1 - |P^{j-1}|) \cdot \{A^{j-1}\} \right\} \quad (12)$$

similarly, $\{A_{MAX}\}$ is calculated from the equation

$$\{A_{MAX}^j\} = \{A^{j-1}\} + \beta \left\{ (1 - |P^{j-1}|) \cdot \{A^{j-1}\} \right\} \quad (13)$$

where β is a scaling factor to adjust the refined domain. Once again, the upper and lower values are checked to insure that the values remain in the feasible domain.

In general, the procedure using the orthogonal array only needs to be carried out once in most practical applications. This results in a reasonably optimum design without exhaustive computations. The final results of this procedure are relatively independent of the starting point in the analysis.

B. STATICALLY INDETERMINATE STAR-SHAPED TRUSS EXAMPLE

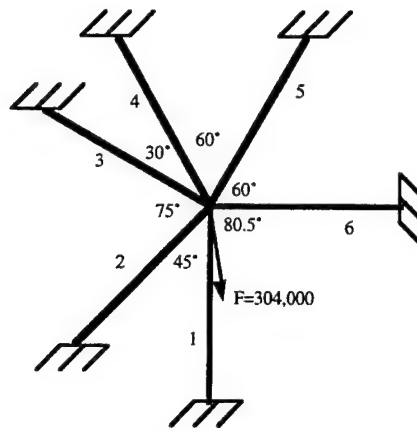


Figure 15. 6-Bar Star-Shaped Truss

Following the procedure outlined in the previous section, this example shows a statically indeterminate six-bar truss structure with discrete design variables. The geometry was chosen to insure that the problem was fully statically indeterminate. The objective here was to minimize the weight of the structure, while not exceeding the failure stress in any of the six members. The length of all 6 members was 10 in. The discrete cross-sectional areas for all six members were 0.25 in^2 , 0.50 in^2 , 1.00 in^2 , 2.00 in^2 , and 2.50 in^2 . These represent the five discrete design variables to be used in the orthogonal array. The $L_{25}(5^6)$ orthogonal array (Taguchi, 1987) was used for this problem. Table 4 shows the FEA results for each run.

Table 4. FEA Results

Run #	P1	P2	P3	P4	P5	P6	WT
1	4.72	-2.60	3.26	4.61	3.57	-1.04	15.00
2	2.79	-1.60	1.85	2.68	2.15	-0.53	27.50
3	1.53	-0.89	1.00	1.46	1.20	-0.27	52.50
4	0.81	-0.48	0.52	0.77	0.63	-0.14	102.50
5	0.65	-0.38	0.42	0.62	0.51	-0.11	127.50
6	1.31	-0.72	0.90	1.27	0.99	-0.29	67.50
7	0.87	-0.42	0.68	0.90	0.62	-0.28	67.50
8	1.18	-1.05	0.33	0.88	1.17	0.30	67.50
9	1.42	-0.89	0.85	1.31	1.15	-0.17	67.50
10	1.65	-0.63	1.48	1.80	1.05	-0.75	67.50
11	1.11	-0.92	0.40	0.87	1.06	0.18	72.50
12	1.41	-0.97	0.75	1.25	1.20	-0.05	72.50
13	0.96	-0.52	0.67	0.94	0.72	-0.22	72.50
14	1.12	-0.14	1.36	1.43	0.51	-0.92	72.50
15	0.96	-0.48	0.72	0.97	0.69	-0.28	72.50
16	0.79	-0.40	0.58	0.79	0.57	-0.22	82.50
17	1.04	-0.83	0.41	0.84	0.96	0.13	82.50
18	0.85	-0.53	0.52	0.79	0.68	-0.11	82.50
19	0.67	-0.27	0.59	0.73	0.44	-0.29	82.50
20	0.95	-0.18	1.07	1.17	0.47	-0.69	82.50
21	0.72	-0.59	0.26	0.57	0.68	0.11	87.50
22	0.61	-0.27	0.51	0.65	0.42	-0.23	87.50
23	0.85	-0.26	0.84	0.98	0.50	-0.48	87.50
24	0.91	-0.48	0.66	0.91	0.67	-0.24	87.50
25	0.72	-0.35	0.55	0.73	0.51	-0.22	87.50

From the results shown in Table 4, the overall sensitivities of the constraints to the design variables $\frac{\Delta P_i}{\Delta A_j}$ between each level, are computed. For each level apply Equation 5 and then compute ΔA_j . If ΔA_j is less than the amount available for the reduction of each design variable, then the value of the design variable can be further reduced.

In this example, the 7th run is used as the initial design and is updated based upon the previously described procedure. Notice that the weight of the 7th run is 67.5 lbs., which ends up being very close to the final value. One could stop here and get a very good feasible design. The final result was

$$\{A^2\}^T = \{0.50 \quad 0.25 \quad 1.00 \quad 2.00 \quad 2.00 \quad 0.25\} \quad (14)$$

$$\{P^2\}^T = \{0.97 \quad -0.58 \quad 0.62 \quad 0.92 \quad 0.76 \quad -0.16\} \quad (15)$$

Because each of the values in the results vector does not violate any of the design constraints, the values in Equation 14 are acceptable design variables. The weight for this structure is 60 lbs., a reduction of 40% over the most conservative design for this structure of 150 lbs in 26 FEM calculations.

C. OPTIMIZATION OF A 12-BAR FRAME STRUCTURE

As discussed earlier, the 12-bar frame structure problem is first solved using common numerical optimization techniques. Next, the problem is solved using the procedures developed in this research. Figure 16 shows the 12-bar frame structure.

1. Solution Using Common Numerical Optimization Techniques

The purpose of this example was to minimize the volume of a frame structure subject to a horizontal load of 1000 lbs at the top of the left edge, and 1000 lbs. at the center of the left edge (See Figure 16). The constraints on the problem included the maximum stress in any member could not exceed the yield strength of the material, the maximum displacement of any node was not to exceed .125", and the lowest natural frequency of the system must be larger than 7 Hz.

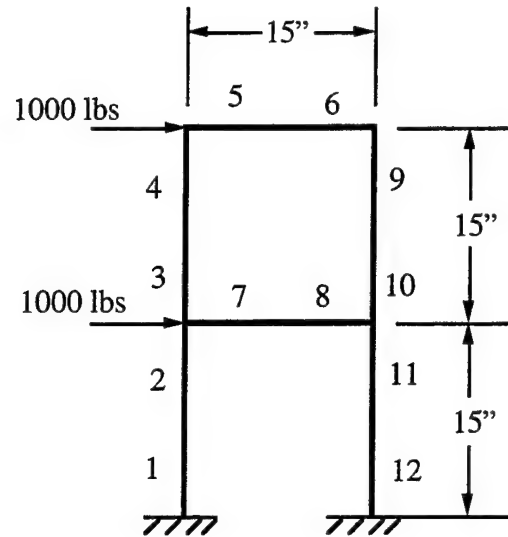


Figure 16. 12-Element Frame Structure

The cross-sectional area of each member was rectangular with a 2:1 height-to-width ratio. Therefore, the heights of each of the members made up the 12 design variables. The upper limit for each member was $h=4.0$ ", and the lower limit for each member was $h=0.1$ ".

The main program executed the MATLAB *constr* optimization routine that called a 'function M-file' (*fun.m*) which calculated the values of the objective and constraints (Grace, 1992). The 'function M-file' also called 2 FEM programs that were used to calculate the constraints. The first FEM program (*disp_frame_fem.m*), calculated the stress in each member and determined the maximum horizontal displacement of any node. The second FEM program (*eig_frame_fem.m*), calculated the natural frequencies of the system.

The problem statement for the constrained optimization problem is given by the following objective function, $F(h)$, and constraints, $g(h)$:

$$\text{minimize:} \quad F(h) = \sum_{i=1}^{12} b_i h_i L \quad (16)$$

$$\text{subject to: } g(i) = \left| \frac{\sigma_{\max}}{S_{\text{yield}}} \right| - 1 \leq 0 \text{ for } i = 1, \dots, 12 \quad (17)$$

$$g(13) = 1 - \frac{\omega_n}{\Omega} \leq 0 \quad (18)$$

$$g(14) = \frac{u_x}{u_{\max}} - 1 \leq 0 \quad (19)$$

$$0.1 \leq h_i \leq 4.0 \text{ for } i = 1, \dots, 12 \quad (20)$$

where: $u_{\max} = 0.125''$

$$\Omega = 7 \text{ Hz}$$

$$S_{\text{yield}} = 78000 \text{ psi}$$

$$b_i = 0.5 h_i$$

h_i are the 12 design variables

The constraints are calculated from FEM calculations

The *constr* function in MATLAB used for this constrained optimization problem, uses a Sequential Quadratic Programming (SQP) method. In this method, a Quadratic Programming subproblem is solved at each iteration, and an estimate of the Hessian matrix of the Lagrangian is updated at each iteration using the BFGS formula. (Grace, 1992) The termination criterion used for this program are as follows:

Termination criteria for $\mathbf{h} = 1 \times 10^{-1}$

Termination criteria for $f = 1 \times 10^{-1}$

Termination criteria for $\mathbf{g} = 1 \times 10^{-2}$

Maximum function calls = 1000

The program was run for 6 different initial values of \mathbf{h}_0 . Table 5 shows the value of the objective function at the optimum in each case. Also shown are the values of the 14

constraints and the 12 design variables corresponding to the optimum. The last row in the table shows the number of function evaluations for each trial. The function evaluation denotes a set of finite element analyses of the frame structure for both static and eigenvalue analyses.

The results show that the design was improved tremendously over the initial design. The objective function for the 4.0 initial design (worst case) was, $F(\mathbf{h}^0) = 720.00 \text{ in}^3$. Compared with the corresponding optimum design of $F(\mathbf{h}^*) = 230.69 \text{ in}^3$.

From the results in Table 5, it can be seen that depending on the starting point, the optimum design goes towards 1 of 2 directions. Initial values of 4.0, 3.9, 1.0, and 0.1 converge towards 1 final design. While, initial values of 3.0 and 2.0 converge towards another final design. The different designs can be characterized by the $h(1)$ and $h(12)$ values. Both designs have essentially the same value of the objective function, however the designs are considerably different. It appears that either the left side or the right side of the frame, but not both, carries the bulk of the load depending on the initial design.

An additional constraint of the buckling load in each compressive member would be useful and might alleviate the problem of the 2 final designs. Also, constraints that restrict the relative displacement of the nodes might prove useful. Further improvement in the convergence of the optimal designs might be achieved by including analytically supplied gradient information to the optimizer, such as partial derivatives of the objective and constraint functions.

Table 5. Optimization Results for 6 Different Initial Design Values

	Starting Values (h_0) for all 12 Design Variables					
	4.0	3.9	3.0	2.0	1.0	0.1
Objective Function	230.6929	230.7695	230.6510	230.9516	230.9296	233.8899
g(1)	-0.8938	-0.8897	-0.8696	-0.8671	-0.8832	-0.8886
g(2)	-0.8983	-0.8973	-0.8363	-0.8393	-0.9035	-0.9023
g(3)	-0.9066	-0.9062	-0.8900	-0.8867	-0.9058	-0.9192
g(4)	-0.8771	-0.8832	-0.9061	-0.9085	-0.8760	-0.8938
g(5)	-0.9610	-0.9611	-0.8775	-0.8823	-0.9622	-0.9818
g(6)	-0.8801	-0.8901	-0.9572	-0.9570	-0.8946	-0.9955
g(7)	-0.9817	-0.9815	-0.9701	-0.9700	-0.9834	-0.9709
g(8)	-0.9664	-0.9649	-0.9844	-0.9846	-0.9723	-0.9108
g(9)	-0.9110	-0.9123	-0.8780	-0.8792	-0.9109	-0.9820
g(10)	-0.9026	-0.8945	-0.9037	-0.9050	-0.8896	-0.9739
g(11)	-0.8369	-0.8393	-0.9488	-0.9524	-0.8381	-0.8374
g(12)	-0.8696	-0.8676	-0.9404	-0.9416	-0.8669	-0.8601
g(13)	-0.4135	-0.4067	-0.4060	-0.4050	-0.4055	0.0005
g(14)	0.0025	0.0001	0.0037	0.0001	0.0008	0.0000
h(1)	1.1282	1.0585	3.7739	3.7482	0.9787	1.0441
h(2)	1.0646	1.0255	2.4621	2.4886	1.0243	1.0466
h(3)	3.1979	3.2091	1.1153	1.0750	3.1925	3.8500
h(4)	2.0968	2.1385	1.5453	1.5359	2.1115	2.7891
h(5)	2.1535	2.1918	1.4336	1.4332	2.2280	0.3848
h(6)	1.4213	1.4161	2.2020	2.1987	1.4467	0.3905
h(7)	2.8990	2.9361	2.3030	2.3172	2.9048	3.4584
h(8)	2.3161	2.3199	2.8901	2.9303	2.3999	2.3434
h(9)	1.5705	1.5275	2.1355	2.1345	1.5303	0.2460
h(10)	1.1639	1.0683	3.1749	3.2037	1.0696	0.1578
h(11)	2.4631	2.4830	1.0405	1.0222	2.4995	2.5613
h(12)	3.7649	3.7474	1.1347	1.0588	3.7331	3.6312
Active Constraints	14	14	14	14	14	13, 14
Function Calls	333	340	285	395	496	857

2. Solution Using Orthogonal Array Optimization Technique

The same problem is solved again using the technique developed in this research. The optimization problem is changed slightly for implementation of this technique. Instead of keeping all of the constraints negative, the normalized stress, displacement, and lowest natural frequency are maintained between 0 and 1, where values of 1 denote active constraints.

Therefore, the problem statement for the constrained optimization problem is given by the following objective function, $F(h)$, and constraints, $g(h)$:

$$\text{minimize: } F(h) = \sum_{i=1}^{12} b_i h_i L \quad (21)$$

$$\text{subject to: } g(i) = \left| \frac{\sigma_{\max}}{S_{\text{yield}}} \right| \leq 1 \text{ for } i = 1, \dots, 12 \quad (22)$$

$$g(13) = \frac{\Omega}{\omega_n} \leq 1 \quad (23)$$

$$g(14) = \frac{u_x}{u_{\max}} \leq 1 \quad (24)$$

$$0.1 \leq h_i \leq 4.0 \text{ for } i = 1, \dots, 12 \quad (25)$$

$$\text{where: } u_{\max} = 0.125''$$

$$\Omega = 7 \text{ Hz}$$

$$S_{\text{yield}} = 78000 \text{ psi}$$

$$b_i = 0.5 h_i$$

h_i are the 12 design variables

The constraints are calculated from FEM calculations

The first step is to select an appropriate orthogonal array to deal with the 12 design variables in this problem. Therefore, the $L_{16}(2^{15})$ Orthogonal Array (Taguchi, 1987) as seen in Table 6 is used.

Table 6. $L_{16}(2^{15})$ Orthogonal Array

$L_{16}(2^{15})$															
Parameter No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of Runs															
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

The values of 1 in the orthogonal array will correspond to $h = 1.00''$, and the values of 2 will correspond to $h = 4.00''$. The FEM programs are then run for the 16 runs in the orthogonal array with the appropriate values of h for each design variable. Since there are only 12 design variables, the last 3 columns could be used for interactions, but will not be used in this example. The results from the running of the orthogonal array are shown in Table 7.

Table 7. Results from the L16(2¹⁵) Orthogonal Array

Run No.	Norm. Stress. Element 1	Norm. Stress. Element 2	Norm. Stress. Element 3	Norm. Stress. Element 4	Norm. Stress. Element 5	Norm. Stress. Element 6	Norm. Stress. Element 7	Norm. Stress. Element 8	Norm. Stress. Element 9	Norm. Stress. Element 10	Norm. Stress. Element 11	Norm. Stress. Element 12	Max. Norm. Displ.	Norm. Min. Natural Freq.	Weight
1	2.6902	2.0322	0.9896	1.3444	-0.0128	-0.0128	-0.0128	-0.0128	-1.3352	-0.9805	-1.9863	-2.6441	38.4095	2.2742	45.00
2	0.1501	0.1827	0.2315	0.1737	-0.0546	-0.0546	-0.4163	-0.0116	-0.0305	-0.0631	-0.1081	-0.1774	1.8670	1.0978	326.25
3	0.8151	0.8439	1.2901	0.0578	0.0279	0.0004	-0.0082	-0.3943	0.0210	0.1493	-2.2303	-0.0963	6.8973	2.9096	326.25
4	0.6602	0.6581	0.1815	0.0086	-0.0083	-0.0200	-0.0126	-0.0516	-0.0454	-0.0196	-0.0963	-2.0859	4.0895	3.0482	495.00
5	1.8663	0.0808	0.0368	0.8278	-0.0133	-0.0006	-0.0354	-2.2470	-0.6759	-0.0151	-0.0275	-0.8989	8.5306	2.9907	382.50
6	1.1958	0.0560	0.0298	0.6499	-0.5278	-0.0088	-0.0270	-0.0270	-0.0258	-0.5912	-1.2226	-0.0573	3.4959	2.1182	438.75
7	0.1583	0.0056	0.0155	0.0324	-0.0123	-0.5064	-0.0591	-0.0591	-0.5064	-0.0191	-0.0960	-0.1737	1.8519	1.4112	438.75
8	2.1311	0.0896	0.0871	0.1003	-0.0330	-2.0618	0.4048	0.0066	-0.0330	-0.8608	-0.8804	-0.8828	8.7463	3.7037	382.50
9	0.0855	1.8414	0.0450	1.0612	-0.0039	-0.2402	-0.0224	-1.4102	-0.0092	-0.2260	-0.0251	-0.7464	6.0302	2.5746	382.50
10	0.0571	1.2030	0.0323	0.7029	-0.0030	-0.1607	-0.0054	-0.0054	-0.5441	-0.0235	-1.1767	-0.0553	3.2592	1.7746	438.75
11	0.0039	0.1415	0.0090	0.0164	-0.7389	-0.0124	-0.0810	-0.0810	-0.0383	-0.8123	-0.0827	-0.1541	2.5589	1.9492	438.75
12	0.1001	2.1992	0.0099	0.0115	-0.2700	-0.0054	1.1727	0.0192	-1.1891	-0.0517	-0.5254	-0.5843	6.9061	2.9060	382.50
13	0.1426	0.0715	0.0543	0.0732	-0.0021	-0.0359	0.0294	0.0294	-0.0708	-0.0353	-0.0146	-0.0429	0.9212	1.1302	382.50
14	0.0780	0.0460	1.1533	1.1723	-0.0008	-0.0008	-0.3639	-0.0064	-1.1679	-1.1506	-0.0426	-0.0822	8.0068	2.8112	438.75
15	0.1309	0.0473	0.6234	0.0165	-0.0297	-0.0297	0.0150	0.8846	-0.0161	-0.0326	0.1851	0.0033	4.4952	2.0479	438.75
16	0.0893	0.0557	1.0978	0.0243	-0.2669	-0.2669	0.0238	0.0222	-0.9740	-1.0773	-0.0052	-0.1114	7.8706	1.9429	382.50

From the FEM results for each run in the orthogonal array, the averaged Jacobian is computed (See Equation 3) which contains the average overall sensitivities of each of the 12 design variables to the 14 constraints. The assumption is made that the sensitivities of the normalized stresses in each member due to other members is small, and therefore neglected, compared to the sensitivities of the normalized stresses due to itself. Therefore the averaged Jacobian matrix is simplified as shown in Table 8.

The absolute maximum value in each column is then used to determine the amount each design variable, h_i , should be changed because the absolute maximum value is the most constraining. It is denoted by $\left(\frac{\Delta P}{\Delta h_j} \right)_{\max}$. Equation 6 will be used to calculate the value Δh_j , where $\Delta h_j = \Delta x_j$.

Table 9 shows the 1-D search for the $h_0=[4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4]$ starting point. The highlighted values are the maximum constraint values (1 is the target). For the case when $\bar{P}_i = 2.2$, the Maximum Normalized Displacement exceeds 1, therefore the distance in the search direction is bracketed and simple interpolation is used to get a value close to 1.

The results for several different starting points are shown in Table 10. The case when $h_0 = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$ is initially in the infeasible domain, with a resulting value in the feasible domain, $h_1=[4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4]$, from which the procedure can be repeated.

Figures 17-20 show the displacements of the frame for the 4 initial designs. The magnification of the displacements for these plots is 50x. Figures 21-23 show the displacements of the final design determined by using the orthogonal array for 3 of the initial conditions.

Figures 24-26 show the first 3 mode shapes of the final design found from the initial design of $h_0=[4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4 \ 4]$. The magnification of displacements is 10x.

Table 8. Average Overall Sensitivities from the L₁₆(2¹⁵) Orthogonal Array

	1	2	3	4	5	6	7	8	9	10	11	12
$\frac{\Delta P_1}{\Delta h_1}$	-0.37											
$\frac{\Delta P_2}{\Delta h_2}$		-0.36										
$\frac{\Delta P_3}{\Delta h_3}$			-0.22									
$\frac{\Delta P_4}{\Delta h_4}$				-0.24								
$\frac{\Delta P_5}{\Delta h_5}$					0.08							
$\frac{\Delta P_6}{\Delta h_6}$						0.14						
$\frac{\Delta P_7}{\Delta h_7}$							-0.03					
$\frac{\Delta P_8}{\Delta h_8}$								0.13				
$\frac{\Delta P_9}{\Delta h_9}$									0.25			
$\frac{\Delta P_{10}}{\Delta h_{10}}$										0.22		
$\frac{\Delta P_{11}}{\Delta h_{11}}$											0.31	
$\frac{\Delta P_{12}}{\Delta h_{12}}$												0.30
$\frac{\Delta P_{13}}{\Delta h_i}$	-1.41	-1.09	-1.30	-1.13	-1.43	-1.30	-1.03	-1.06	-2.06	-2.09	-1.35	-2.04
$\frac{\Delta P_{14}}{\Delta h_i}$	-0.10	-0.02	0.09	0.13	0.08	0.12	0.09	0.09	-0.06	-0.16	-0.04	-0.19

where: $\frac{\Delta P_1}{\Delta h_1}$ = the sensitivity of the stress in element 1 to height 1
 $\frac{\Delta P_{13}}{\Delta h_i}$ = the sensitivity of the maximum displacement to height i
 $\frac{\Delta P_{14}}{\Delta h_i}$ = the sensitivity of the minimum natural frequency to height i

for $i = 1$ through 12

Table 9. Optimization Results Using Orthogonal Array with $h_0=[4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ 4]$

	Function Evaluations	Maximum Stress Constraint	Minimum Normalized Natural Frequency	Maximum Normalized Displacement	Weight
Orthogonal Array	16				
h_0	1				
$\bar{P}_i = 0.9$	1	0.0677	0.6804	0.2898	537.96
$\bar{P}_i = 1.2$	1	0.0802	0.7295	0.3784	471.93
$\bar{P}_i = 1.4$	1	0.0903	0.7672	0.4579	430.50
$\bar{P}_i = 2.0$	1	-0.1348	0.9144	0.8748	318.63
$\bar{P}_i = 2.2$	1	-0.1573	0.9795	1.1195	285.49
$\bar{P}_i = 2.1$	1	0.1455	0.9457	0.9874	301.80

Figures 27-29 show the first 3 mode shapes of the final design found from the initial design of $h_0=[3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3]$. The magnification of displacements is 10x.

Figures 30-32 show the first 3 mode shapes of the final design found from the initial design of $h_0=[2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2\ 2]$. The magnification of displacements is 10x.

As shown, the averaged Jacobian is a matrix of the average overall sensitivities which yields a search direction that, when coupled with a 1-D search technique, can determine a design which lowers the objective function and always remains in the feasible domain. The starting point value X^{q-1} is updated according to

$$\bar{X}^q = \bar{X}^{q-1} + \alpha^* \bar{S}^q \quad (26)$$

where X^q is the new vector of design variables, q is the iteration number, S is a search direction vector (here calculated by the averaged Jacobian matrix), and the scalar quantity α is the distance in the search direction, S .

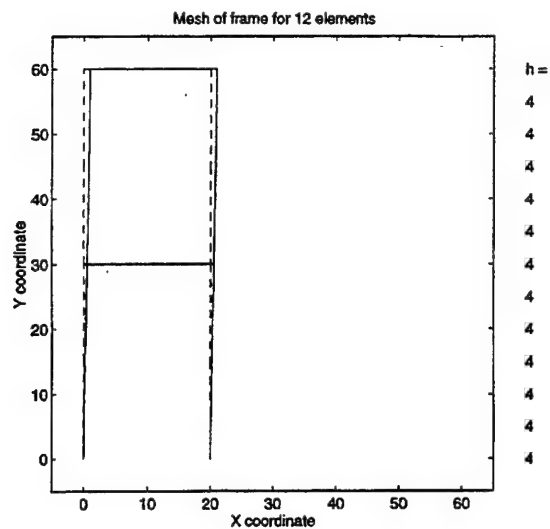


Figure 17. Displacement for Initial Design, $h=4.0$, Max. Disp. = 0.0195" (Mag = 50x)

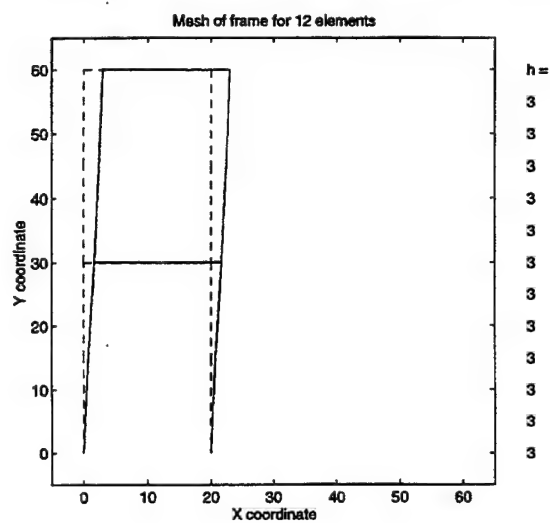


Figure 18. Displacement for Initial Design, $h=3.0$, Max. Disp. = 0.0595" (Mag = 50x)

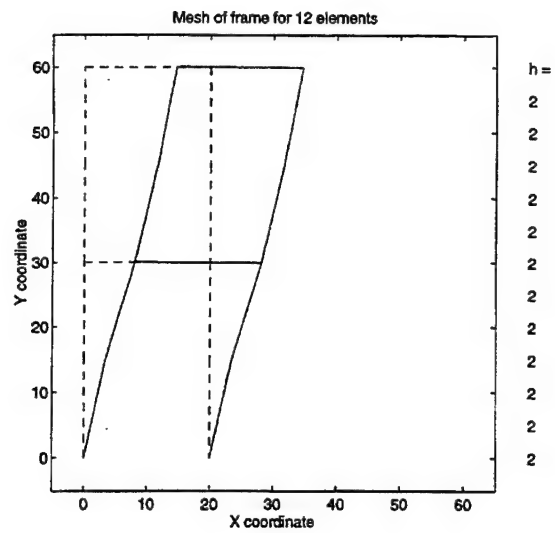


Figure 19. Displacement for Initial Design, $h=2.0$, Max. Disp. = 0.2930" (Mag = 50x)

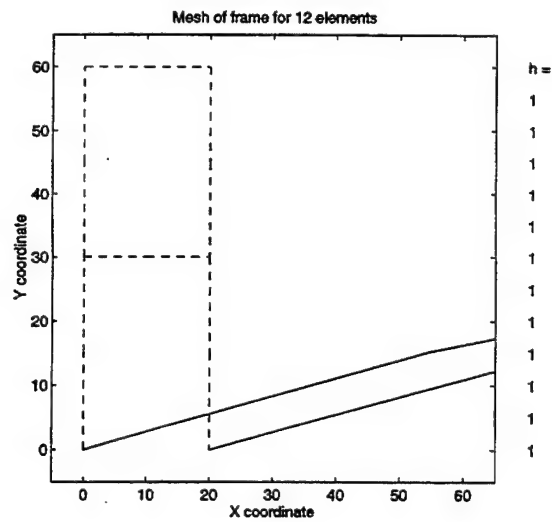


Figure 20. Displacement for Initial Design, $h=1.0$, Max. Disp. = 4.6091" (Mag = 50x)

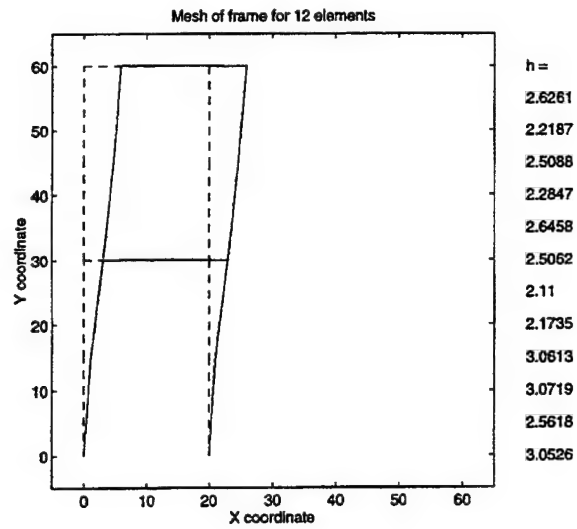


Figure 21. Displacement for Final Design, $h_0=4.0$, Max. Disp. = 0.1185" (Mag = 50x)

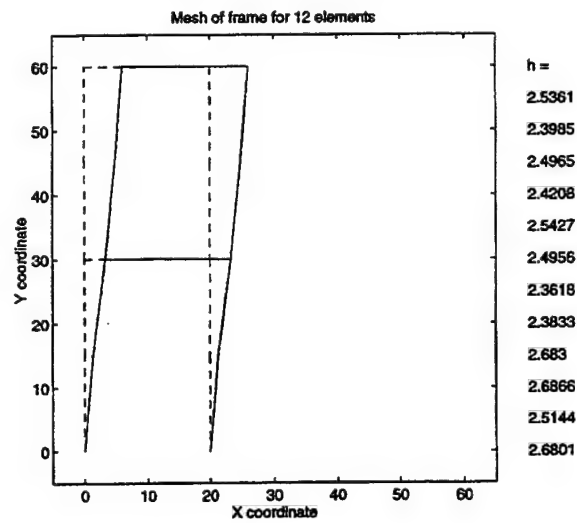


Figure 22. Displacement for Final Design, $h_0=3.0$, Max. Disp. = 0.1206" (Mag = 50x)

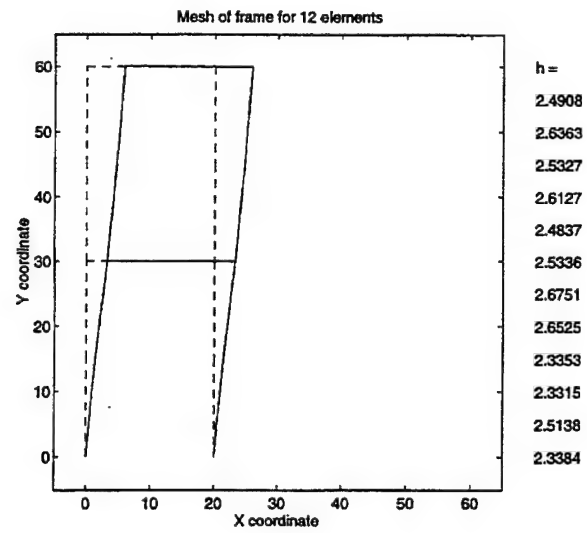


Figure 23. Displacement for Final Design, $h_0=2.0$, Max. Disp. = 0.1185" (Mag = 50x)

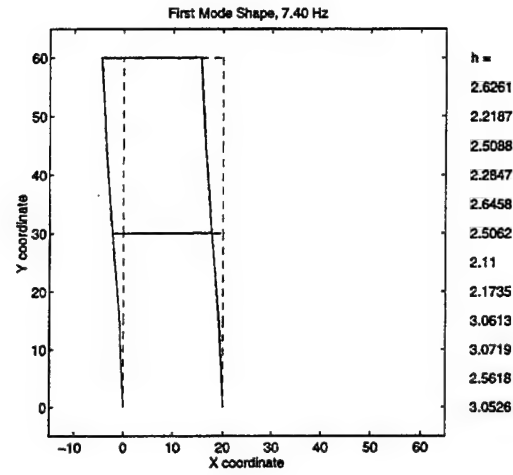


Figure 24. 1st Mode Shape with $h_0=4.0$ (Mag = 10x)

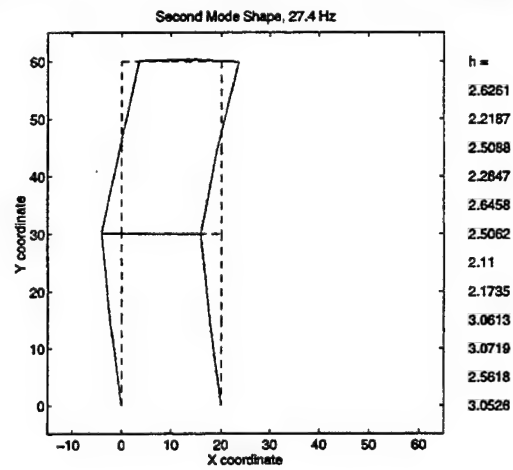


Figure 25. 2nd Mode Shape with $h_0=4.0$ (Mag = 10x)

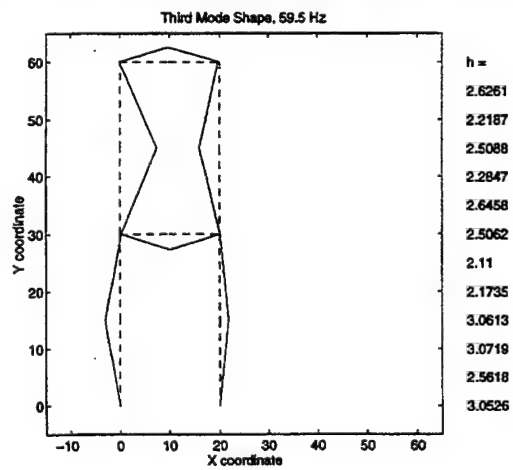


Figure 26. 3rd Mode Shape with $h_0=4.0$ (Mag = 10x)

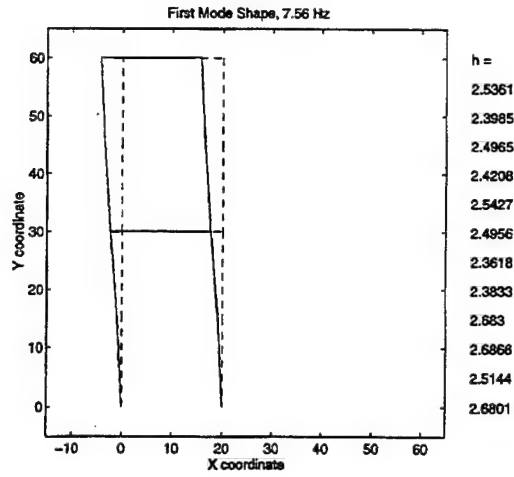


Figure 27. 1st Mode Shape with $h_0=3.0$ (Mag = 10x)

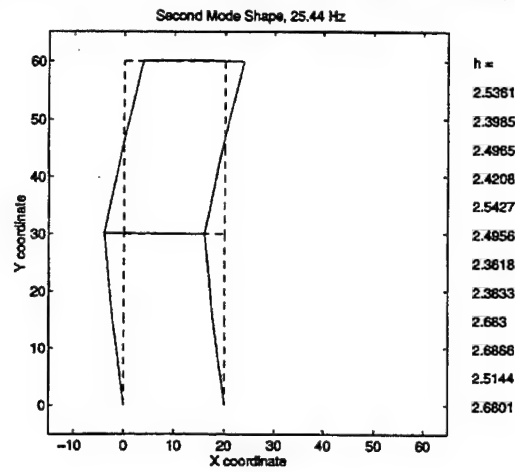


Figure 28. 2nd Mode Shape with $h_0=3.0$ (Mag = 10x)

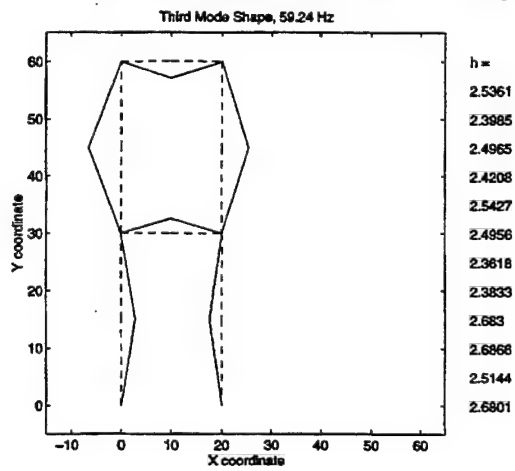


Figure 29. 3rd Mode Shape with $h_0=3.0$ (Mag = 10x)

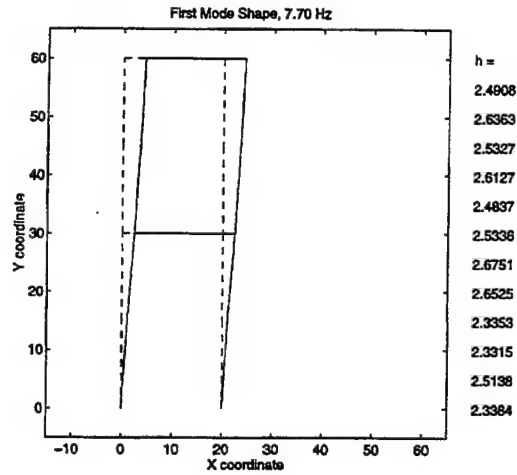


Figure 30. 1st Mode Shape with $h_0=2$ (Mag = 10)

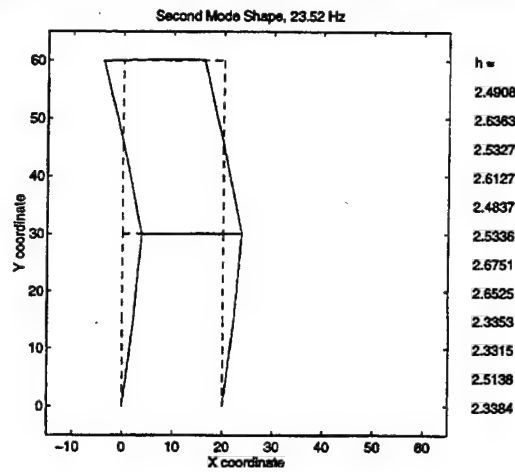


Figure 31. 2nd Mode Shape with $h_0=2$ (Mag = 10)

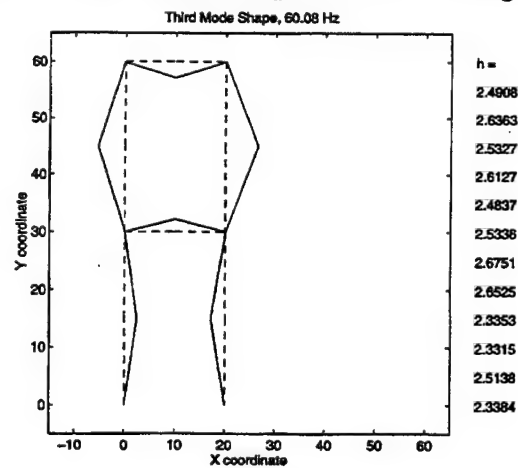


Figure 32. 3rd Mode Shape with $h_0=2$, (Mag = 10)

Table 10. Optimization Results (Orthogonal Array) for 4 Different Initial Designs

	Starting Values (h_0) for all 12 Design Variables			
	$h_0 = 4.0$	$h_0 = 3.0$	$h_0 = 2.0$	$h_0 = 1.0$
Objective Function	301.8008	285.57	284.45	720.00
g(1)	0.1407	0.1685	0.1852	0.0461
g(2)	0.1156	0.1317	0.1334	0.0343
g(3)	0.0609	0.0640	0.0670	0.0167
g(4)	0.1101	0.0946	0.0796	0.0217
g(5)	-0.0148	-0.0067	-0.0075	-0.0008
g(6)	-0.0173	-0.0070	-0.0071	-0.0008
g(7)	-0.0182	-0.0083	-0.0084	-0.0008
g(8)	-0.0168	-0.0081	-0.0086	-0.0008
g(9)	-0.0619	-0.0764	-0.0960	-0.0212
g(10)	-0.0252	-0.0488	-0.0802	-0.0162
g(11)	-0.1001	-0.1198	-0.1317	-0.0314
g(12)	-0.1455	-0.1643	-0.1762	-0.0432
g(13)	0.9457	0.9264	0.9091	0.5872
g(14)	0.9874	1.005	0.9878	0.1629
h(1)	2.6261	2.5361	2.4908	4.0
h(2)	2.2187	2.3985	2.6363	4.0
h(3)	2.5088	2.4965	2.5327	4.0
h(4)	2.2847	2.4208	2.6127	4.0
h(5)	2.6458	2.5427	2.4837	4.0
h(6)	2.5062	2.4956	2.5336	4.0
h(7)	2.1100	2.3618	2.6751	4.0
h(8)	2.1735	2.3833	2.6525	4.0
h(9)	3.0613	2.6830	2.3353	4.0
h(10)	3.0719	2.6866	2.3315	4.0
h(11)	2.5618	2.5144	2.5138	4.0
h(12)	3.0526	2.6801	2.3384	4.0
Active Constraints	14	14	14	N/A
Function Calls	23	22	23	17
Final \bar{P}_i	2.1	1.15	1.75	0.9

In this example, a simple 1-D search algorithm is used which extrapolates out in the direction chosen until a constraint is violated. Once the constraint is violated, the solution is bounded and an interpolation procedure similar to the golden section method may be used.

3. Comparison of the two Optimization Techniques

Table 11 shows a comparison of the convergence rates between the SQP optimization method and the Orthogonal Array Technique. It is important to notice that the SQP routine does not always approach the optimum from the feasible domain. However, the Orthogonal Array Technique always approaches the optimum solution from the feasible domain. This is an extremely important feature of the technique.

Table 11. Convergence Comparison

	SQP			Orthogonal Array		
	Function Evaluations	Objective	Max Constraint Value ≤ 0	Function Evaluations	Objective	Max Constraint Value ≤ 1
$h_0 = \{4.0\}$	28	444.596	-0.5136	23	301.801	.9874
	43	326.061	-0.2585			
	58	274.131	-0.554			
$h_0 = \{3.0\}$	13	405.000	-0.2962	22	285.57	1.005
	28	300.724	-0.2604			
	44	283.249	-0.1643			
$h_0 = \{2.0\}$	13	180	1.344 *	23	284.45	0.9878
	28	158	1.945 *			
	351	230.875	0.0013			
$h_0 = \{1.0\}$	13	45	35.871 *	17	720	0.5872
	496	230.930	.0008	40 (17+23)	301.801	.9874

* indicates infeasible design

For the $h_0 = \{2.0\}$ initial condition, the SQP converges to an optimum of 230.9 in 351 function evaluations (with infeasible solutions at intermediate steps), and the Orthogonal Array method converges to 284.5 in only 23 function evaluations. The Orthogonal Array method converges to within 23% of the optimum design in only 6.5% of the number of function evaluations. This is a tremendous computational savings considering the FEM analyses required at each function evaluation. Further research is

being conducted to improve the convergence closer to the optimum, but studies so far have been inconclusive.

V. CONCLUSION

Improving design in engineering is a critical process in order for businesses to stay competitive and profitable. Therefore, optimization techniques are required to improve the design process and help the engineer to develop better products in the future. This thesis has described a new design optimization technique that couples the orthogonal array experimental procedure with finite element analysis in an effort to develop an economical and efficient way of optimizing engineering designs. This paper has also shown some examples of the procedure which produced very good results. However, the proposed design optimization technique needs to be further developed and refined for applications to various design problems. Results also show that the orthogonal array technique converged towards the optimum at a faster rate than the SQP method for the frame example. Additionally, the goal of reducing the number of FEM calculations was achieved as well as providing the engineer with insight into the effects of changing the design variables on the constraints. This sensitivity information may prove very useful when developing future designs of a similar nature.

VI. RECOMMENDATIONS

Further coupling of the orthogonal array technique with one-dimensional search techniques will provide for a more robust optimization algorithm and should be the topic of future research. In the 1-D search technique a possible technique might be to vary each value of the target value individually, in an effort to further reduce the objective function with a single application of the orthogonal array.

Also, additional studies should look at the problem of refining the solution further towards the actual optimum. One possible method of doing this might be to incorporate a second orthogonal array around a refined domain.

Studies should be conducted on additional highly non-linear problems to further evaluate the performance of this technique.

The cases shown in this thesis, show designs that mostly start from the feasible domain. Additional research should be conducted for cases of an initially infeasible design.

Additionally, utilizing all of the average overall sensitivities and not just the diagonal terms in the first n rows of the averaged Jacobian should be examined to enhance the procedure developed in this research.

Finally, research conducted by Yurkovich (1994) suggests a similar approach to the optimization of structures with the utilization of Taguchi Methods. However, Yurkovich used Taguchi Methods to select the critical design variables and then implemented a conventional optimization technique.

From Yurkovich's research, the incorporation of interactions into the procedure developed in this thesis might prove very useful in improving the optimization technique. Additionally, orthogonal arrays can be further utilized to reduce the number of design variables and interactions that must be analyzed with the finite element method.

LIST OF REFERENCES

- Ealey, Lance A. 1988. *Quality by Design*. Dearborn, MI: ASI Press.
- Grace, Andrew. 1992. *Optimization Toolbox User's Guide* Natick, MA: The MathWorks, Inc.
- Kwon, Young W. and Hyochoong Bang. 1997. *The Finite Element Method using MATLAB*. Boca Raton, FL: CRC Publications.
- Lang, Serge. 1968. *Calculus*. New York: Addison-Wesley.
- Peace, Glen S. 1993. *Taguchi Methods: A Hands-On Approach To Quality Engineering*. Reading, MA: Addison-Wesley.
- Phadke, Madhav S. 1989. *Quality Engineering using Robust Design*. Englewood Cliffs, NJ: Prentice-Hall.
- Ryan, Nancy E., ed. 1988. *Taguchi Methods and QFD*. Dearborn, MI: ASI Press.
- Taguchi, G. and Konishi, S. 1987. *Taguchi Methods: Orthogonal Arrays and Linear Graphs*. Dearborn, MI: ASI Press.
- Vanderplaats, Garret N. 1984. *Numerical Optimization Techniques for Engineering Design: With Applications*. New York: McGraw-Hill.
- Yurkovich, Rudy. 1994. *The use of Taguchi Techniques with the ASTROS Code for Optimum Wing Structural Design*. AIAA Paper 94-1484-CP, 1994 Structures, Structural Dynamics, and Materials Conference.

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